

UNIVERSITY OF OSLO  
Faculty of Mathematics and Natural Sciences

Exam AST2110 — The Universe

Date of exam: Tuesday 8 June 2004

Time for exam: 14.30 – 17.30

This exercise set contains 3 pages.

Attachments: None

Allowed sources of help: Rottmann: “Matematisk formelsamling”

Øgrim og Lian: “Størrelser og enheter i fysikk og teknikk”

Calculator

Two A4 pages (can be written on both sides) with your own notes

*Check that the exercise set is complete before you start solving the problems.*

Exercise 1

- a) The centre of mass reference frame for a system of  $N$  particles is defined by that observed in it,  $\sum_{i=1}^N \mathbf{p}_i = 0$ .  $P_\mu =$

$(E/c, \mathbf{p})$  is a four-vector. Show that for the whole system, observed in the centre of mass frame of reference,  $P_\mu^{CM} = (E_0/c, 0)$ . Express  $P^\mu P_\mu = -E^2/c^2 + p^2$  for the whole system observed in an arbitrary inertial frame of reference by  $E_0$ .

Let us now look at two photons which in the laboratory frame of reference have frequencies  $\nu_1$  and  $\nu_2$ , and in the laboratory frame the angle between their directions of propagation is  $\theta$ .

- b) Show that in the centre of mass reference frame, the total energy of the system is  $E_0 = 2h\sqrt{\nu_1\nu_2} \sin \frac{\theta}{2}$  (remember that  $\sqrt{(1 - \cos \theta)/2} = \sin \frac{\theta}{2}$ ).
- c) What is the frequency of each of the photons seen in the centre of mass reference frame?
- c) Is it possible to find a centre of mass reference frame for this system for all values of  $\theta$ ?

## Exercise 2

- a) Show that a Solar System body at distance  $r$  from the Sun and with an albedo  $a$  under certain assumptions will reach an equilibrium temperature  $T_p = (R_\odot/2r)^{1/2}(1 - a)^{1/4}T_\odot$ , where  $R_\odot$  is the radius of the Sun and  $T_\odot$  is the surface

temperature of the Sun. Explain the most important assumption that must be made. In the rest of this exercise, assume that  $R_{\odot} = 696000\text{km}$  and  $T_{\odot} = 5800\text{K}$ .

- b) For the Earth,  $a = 0.3$  and  $r = 150$  million km. Compare the temperature you get using the equation in exercise a with the average surface temperature of the Earth. What is the main reason for the difference?
- c) Neptune's average distance from the Sun is  $4.5 \times 10^9$  km and its albedo is 0.29. Its surface temperature is 59K. How does this agree with the surface temperature you get? What is the main reason for the difference?

### Exercise 3

- a) Draw Hubble's tuning fork diagram and show where in it the different galaxy types are situated.
- b) What is meant by that the surface brightness in elliptical galaxies and in the bulge of spiral galaxies follow de Vaucouleurs'  $r^{1/4}$  law?
- c) How can the Tully-Fisher relation be used for determining the distances to spiral galaxies?
- d) Express the redshift of a cosmological object by the scale factor of the Universe,  $R$ .